"UNIT - IV"

"MATRICES,"

Date: 30/11/22
P. No.:

★ **Matrix** :- A system of mn number (real or Complex) arranged in a rectangular array of m rows & n columns is called ~~of~~ a matrix of order $m \times n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & \cdots & & \\ \vdots & & & \\ a_{m1} & & & \end{bmatrix}$$

$rt - s^2$

$\left(\frac{+1}{9}\right) \times \left(\frac{+1}{8}\right) - \left(\frac{8 \times 8}{27 \times 27}\right)$

$\frac{1}{72} - \frac{64}{729} =$ ______

$(27)^2$

0749
28
729

Date :
P. No. :

★ Elementary Row operations :- The elementary rows operations are as follows:—

① Interchange of any two rows of a matrix generally the interchange i-th & j-th is denoted by $R_{ij}$.

② The multiplication of every element of any row by a non-zero constant, generally the multi. of every element of i-th row by a constant $k \neq 0$, is generally denoted by $R_i(k)$.

③ The addition to the element of a row, the product of the corresponding element of any other row, by any non-zero constant generally the addition to the element of i-th row the product of corresponding elements of j-th row, by a constant $(k) \neq 0$, and is denoted by $R_{ij}(k)$.

eg. → ① $R_1 \rightarrow R_1 + R_2(2)$
$R_{ij}(k) \longrightarrow R_{12}(2)$

② $R_1 \longrightarrow R_1 - 2R_2$
$R_{ij}(k) \longrightarrow R_{12}(-2)$

③ $R_1 \leftrightarrow R_2 \Rightarrow R_{12}$

## * Minor of a matrix :-

Let ~~a, b be any~~ A be a matrix rectangular or square from this matrix A delete all columns or rows leaving a certain p columns & p rows. Now if $p > 1$, then the elements which have been left consitute a square matrix of order p. The determinant of this square matrix is called minor of (A) of order p.

## * Rank of a Matrix :-

Let A be any matrix then no. r is called the rank of the matrix. If it obeys to following 2 properties —

1. There is atleast 1 minor of A of order (r). which does not vanish.
2. Every minor of A of order higher than are vanish.

# Note →

1. Rank of zero matrix is 0.
2. Rank of unit matrix ($I_n$) is n.
3. The rank of non-singular matrix (A) of order (n) is n.

Since $|A| \neq 0$.

4. The rank of matrix (A) is denoted by symbol $\rho(A)$.

★ NOTE :→ Rank of any matrix $\le$ No. of rows

- Nullity of a matrix :- Let A be a square matrix of order (n) and if the rank of (A) matrix is $r$, the $n-r$ is called nullity of a matrix. ~~That~~ denoted by $N(A)$.

$$\boxed{N(A) = n - r}$$

**Que.** Find the rank and nullity of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

$$C_3 \to C_3 - (C_1 + C_2)$$

$$\begin{bmatrix} 1 & 2 & 2 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 3 & 5 & 0 \end{bmatrix}$$

[Non-zero rows $\underset{\text{utni}}{=}$ Rank hogi]

→ Rank find krna ka best form!

Mark "IMP"

★ Echelon form :-

① A matrix is called Echelon form. If all the non-zero rows, if any preceed the zero rows.

② The no. of zeroes preceeding the first non-zero element in a row is less than the no. of such zeroes in the next row.

③ The first non-zero element in each row is unity.

Que.1|

$$A = \begin{bmatrix} 1 & 5 & 6 & 7 & 0 & 8 \\ 0 & 1 & 2 & 5 & -1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Date :
P. No. :

• Normal form :- If A is the matrix of order $m \times n$ and rank $r$ then a can be reduced by the no. of elementary transformation. to any of the following form.

(i) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} I_r & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ (iv) $[I_r]$

Que $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ Find the rank & nullity of the following matrix.

$R_{31}(-1)$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

$R_{21}(-3)$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

$R_{32}$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2\left(-\frac{1}{2}\right)$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Nullity of matrix $= n - r$
$= 3 - 2$
$= 1$ Ans.

n = order of matrix

r = rank

$\rho(A) = 2$

$\boxed{\text{Rank} = 2}$

6 marks *IMP

**Que** Find the normal form of the matrix (A) and hence find its rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$$R_{12} \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \to R_2 + R_1(-2)$

$R_3 \to R_3 + R_1(-3)$

$R_4 \to R_4 + R_1(-6)$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_4 \to R_4 + R_3(-1)$

$R_4 \to R_4 + R_2(-1)$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_{21}(1) = C_2 \to C_2 + C_1(1)$

$C_{31}(2) = C_3 \to C_3 + C_1(2)$

$C_{41}(4) = C_4 \to C_4 + C_1(4)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_{23}(-1) = R_2 \to R_2 + (-1)R_3$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_{32}(-4) = R_3 \to R_3 + R_2(-4)$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_{32}(6) = C_3 \to C_3 + C_2(6)$

$C_{42}(3) = C_4 \to C_4 + C_2(3)$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3\left(\frac{1}{33}\right) - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_{43}(-22) = C_4 \to C_4 + C_3(-22)$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$ $\boxed{\rho(A) = 3}$ Rank

# • MATHS ASSIGNMENT QUESTIONS (UNIT-4)

① Find the normal form of the matrix (A) and hence find its rank.

$$A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

**Que ②** Find the normal form of the matrix (A) and hence find its rank. $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$.

**Que ③** ~~Find that~~ For what value of $\lambda$ & $u$, the eq. are –

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = 4$

(i) no sol.$^n$.
(ii) a unique sol.$^n$
(iii) Infinite many sol.$^n$

**Que 4** $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ If matrix A = then verify Cayley-Hamilton theoram Hence find $A^{-1}$.

**Que 5** $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ If matrix A = then satifies cayley-Hamilton theoram Hence find $A^{-1}$.

★ Solution of simultaneous eq$^n$ by elementary Transformation and consistency of the eqns:-

$a_{11}x + a_{12}y + a_{13}z + \ldots = b_1$

$a_{21}x + a_{22}y + a_{23}z + \ldots = b_2$

$AX = B$

$X = A^{-1}B$

unknown

$\rho[A \vdots B] = \rho(A)$

Augmented

Consistant

$\boxed{\rho[A \vdots B] = \rho(A)}$

Rank

$\boxed{\rho(A \vdots B) \neq \rho[A]}$ Inconsistant

• Working Rule of Upper topics:-

First of all, right the augmented matrix $(A \vdots B)$. Now reduce $A \vdots B$ to the echlon form by the application of elementary row operation only it gives the rank of $A \vdots B$. Now obtain $\rho(A)$ after deleting last coloum from the echelon's form of $(A \vdots B)$. It gives the rank of $(A)$.

Now Following two cases arrived.

(1) Rank of A ≠ Rank of (A ⋮ B) –
In this position the eq.$^n$ are called Incosistent, then they don't have no sol.$^n$.

(2) Rank of A = Rank of (A ⋮ B) –
In this position, the eq.$^n$ are called consistent, they have solution

★ Now following sub-cases are arrived –

(1) If $r = n$ – ($r$ is rank & $n$ is unknowns)
($n \rightarrow$ no. of unknown ~~value of x, y, z~~)
In this case, the eq.$^n$ have unique solution.

(2) If $r < n$, $\rightarrow$ In this case, the eq.$^n$ have Infinite solution.

Date :
P. No. :

Que. Examine the following eq.$^n$ for consistency.

$$5x + 3y + 14z = 4$$
$$y + 2z = 1$$
$$x - y + 2z = 0$$
$$2x + y + 6z = 2$$

Rows mai only operation lagege
columns mai nhi

convert → eschelon form's

$$AX = B$$

$$[A:B] = \left[\begin{array}{ccc:c} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & 1 & 6 & 2 \end{array}\right]$$

$$R_{31} \left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4 \\ 2 & 1 & 6 & 2 \end{array}\right]$$

$R_4 \to R_4 + R_1(-2)$

$\Rightarrow R_3 \to R_3 + R_1(-5)$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \\ 0 & 3 & 2 & 2 \end{array}\right]$$

$R_{32}(-8) = R_3 \to R_3 + R_2(-8)$

$R_{42}(-3) = R_4 \to R_4 + R_2(-3)$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -12 & -4 \\ 0 & 0 & -4 & -1 \end{array}\right]$$

$R_3 \to R_3 + R_4(-3)$

$$\left[\begin{array}{ccc:c} 0 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1/4 \end{array}\right]$$

$\Rightarrow$

$R_{34} \Rightarrow R_3 \leftrightarrow R_3 + R_4$

$R_4(-1)$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1/4 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\boxed{\rho(A:B) = 4}$

$\boxed{\rho(A) = 3}$

$\boxed{\rho(A:B) \neq \rho(A)}$ Inconsistant

Date :
P. No. :

**Que.** Examine the consistency for following eq$^n$, if consistent. Find the complete solution.

$$x+2y-z=3$$
$$3x-y+2z=1$$
$$2x-2y+3z=2$$
$$x-y+z=-1$$

$[A:B]$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$R_{21}(-3) \Rightarrow R_2 \to R_2 + R_1(-3)$
$R_{31}(-2) \Rightarrow R_3 \to R_3 + R_1(-2)$
$R_{41}(-1) \Rightarrow R_4 \to R_4 + R_1(-1)$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right]$$

$R_{23}(-1)$
$R_2(-1)$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right] \Longrightarrow R_{32}(6),\ R_{42}(3) \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 2 & 8 \end{array}\right]$$

$$R_3\left(\frac{1}{5}\right) \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & 8 \end{array}\right] \Longrightarrow R_{43}(-2) \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\rho(A:B) = \rho(A) = 3}$$

Consistant

$\boxed{r=n}$, $3=3$, It has unique sol.$^n$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$$

$$\left\{\begin{array}{l} x+2y-z=3 \\ y=4 \\ z=4 \\ x=-1 \end{array}\right.$$

Que. Test the consistenly & solve:

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$
$$7x+2y+10z=5$$

$$[A : B] = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$$R_{12}(-2) \left[\begin{array}{ccc|c} -1 & -49 & 3 & -14 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$$\begin{array}{l} R_2(-3) \\ R_3(-7) \end{array} \left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & 121 & 11 & -33 \\ 0 & -341 & 31 & -93 \end{array}\right]$$

$$\begin{array}{l} R_2\left(\frac{1}{11}\right) \\ R_3\left(\frac{-1}{31}\right) \end{array} \left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & -11 & 1 & -3 \\ 0 & 11 & -1 & +3 \end{array}\right] \Rightarrow R_{32}(1) \left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & -11 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_2\left(\frac{-1}{11}\right) \left[\begin{array}{ccc|c} 1 & 49 & -3 & 14 \\ 0 & 1 & -1/11 & 3/11 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\rho(A:B) = \rho(A) = 2}$$

it is Consistent

Here, $\boxed{r<n}$ it has infinite sol$^n$

$$\begin{bmatrix} 1 & 49 & -3 \\ 0 & 1 & -1/11 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 3/11 \\ 0 \end{bmatrix}$$

$$x+49y-3z=14$$
$$y-\frac{1}{11}z=\frac{3}{11}$$
$$y=\frac{3}{11}+\frac{1}{11}z$$

Take $(z = k)$ (Constant)

$$y = \frac{3}{11} + \frac{1}{11}k = \frac{3+k}{11} \Rightarrow \boxed{x = \frac{7-16k}{11}}$$

★ **Eigen Values and eigen vector :-**

The eq.$^n$ $|A - \lambda I| = 0$ is called Characteristics eq.$^n$ of A, The roots of this eq.$^n$ are called "roots or eigen values of matrix A.

Characteristics vector / eigen vector : If $\lambda = \lambda_1$ is char. root of 'A' then the non-zero sol.$^n$ of $x = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$ of the eq.$^n$. $(A - \lambda_1 I)X = 0$ is said to be eigen vector of 'A' corresponding to charac. root $\lambda = \lambda_1$.

Que/ Find char. root and eigen vector $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \Rightarrow \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 4$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - \lambda - 6\lambda + 6 = 0$$

$$\lambda(\lambda - 1) - 6(\lambda - 1) = 0$$

$$\boxed{\lambda = 1, 6}$$

# Eigen value of A $\Rightarrow$ 1 and 6

Date :
P. No. :

Eigen vector corresponding the eigen value!

$\boxed{\lambda = 1}$

$(A - \lambda_1 I)x = 0$

$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$x_1 + x_2 = 0$

$x_1 = -x_2$

$\boxed{\frac{x_1}{x_2} = -1}$

For $\lambda = 6$

$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \longrightarrow 4x_2 - x_1 = 0$

$\cancel{-4x_2 + x_1 = 0}$

$4x_2 = x_1$

$\boxed{\frac{x_2}{x_1}} = \frac{1}{4}$

$\boxed{X = \begin{bmatrix} 1 \\ 4 \end{bmatrix}}$

Que. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ Find characteristics root and eigen vector.

$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix}$

$\begin{bmatrix} 6-\lambda & -2 & -2 \\ 0 & 2-\lambda & 2-\lambda \\ 2 & 1 & 3-\lambda \end{bmatrix}$

$(6-\lambda)(6-5\lambda+\lambda^2+2-\lambda)$
$+ 2(-4+2\lambda-4+2\lambda)$

$6\lambda^2 - 36\lambda + 48 - \lambda^3 + 6\lambda^2 \cancel{-8\lambda} \cancel{+8\lambda} - 16$

$-\lambda^3 + 12\lambda^2 - 36\lambda + 32$

$\boxed{\lambda^3 - 12\lambda^2 + 36\lambda - 32}$

Date: 14/12/22
P. No.:

Maths,

Assign.
Que. 1

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Find characteristic, root and eign vector.

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$8-\lambda\left[(7-\lambda)(3-\lambda) - (-4)\times(-4)\right] + 6\left[-6(3-\lambda) - (-4)\times 2\right]$$
$$+ 2\left[24 - (14 - 2\lambda)\right]$$

$$8-\lambda\left[21 - 10\lambda + \lambda^2 - 16\right] + 6\left[-18 + 6\lambda + 8\right]$$
$$+ 2\left[24 - 14 + 2\lambda\right]$$

$$(8-\lambda)\left[\lambda^2 - 10\lambda + 5\right] + (36\lambda - 60) + 20 + 4\lambda$$

$$8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 60 + 20 \cdot$$

$$\boxed{-\lambda^3 + 18\lambda^2 - 45\lambda = 0}$$

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$
$$-\lambda(\lambda^2 - 18\lambda + 45) = 0$$
$$-\lambda[\lambda^2 - 15\lambda - 3\lambda + 45] = 0$$
$$-\lambda[\lambda(\lambda-15) - 3(\lambda-15)] = 0$$
$$-\lambda[(\lambda-3)(\lambda-15)] = 0$$
$$\boxed{\lambda = 0, 3, 15}$$ Ans.

$8+9$

$17$

$$\boxed{\lambda = 0}$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 4 & -3 & 1 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} ① & 4x_1 - 3x_2 + x_3 \\ ② & -6x_1 + 7x_2 - 4x_3 \\ ③ & 2x_1 - 4x_2 + 3x_3 \end{bmatrix} = 0$$

① + ③

$$6x_1 - 7x_2 + 4x_3$$
$$-6x_1 + 7x_2 - 4x_3 \text{ — ②}$$

$$\begin{bmatrix} 4 & -3 & 1 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

② + ③ × 3

$$-6x_1 + 7x_2 - 4x_3 = 0$$
$$6x_1 - 12x_2 + 9x_3 = 0$$

$$-5x_2 + 5x_3 = 0$$
$$5x_2 = 5x_3$$
$$\boxed{x_2 = x_3}$$

eq. ① $[x_2 = x_3]$

$$4x_1 - 3x_2 + x_3 = 0$$
$$4x_1 - 3x_2 + x_2 = 0$$
$$4x_1 - 2x_2 = 0$$

~~eq. ③ × 2 [$x_2 = x_3$]~~

$$4x_1 - 2x_2 = 0$$
$$4x_1 = 2x_2$$
$$\boxed{x_1 = \frac{x_2}{2}}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 1 \end{bmatrix}$$

easiest

★ Cayley-Hamilton theoram:- Every square matrix satisfied its character. eq.$^n$ → $|A - \lambda I| = 0$.

$$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_n I = 0$$

then the eq.$^n$,

put $\lambda = A$

$$① \quad \boxed{A^n + a_1 A^{n-1} + a_2 A^{n-2} + \dots + a_{n-1}A + a_n I = 0}$$

Multiply eq. ① by $A^{-1}$, we get

$$A^{n-1} + a_1 A^{n-2} + \dots + a_{n-1} I + a_n A^{-1} = 0$$

★★★

$$\boxed{A^{-1} = \frac{-1}{a_n} \left[ A^{n-1} + a_1 A^{n-2} + \dots + a_{n-1} \right.}$$

$A^2 = A \cdot A$

$A^3 = A^2 \cdot A$

$A^4 = A^3 \cdot A$

$\vdots$

$A^n = A^{n-1} \cdot A$

Que Find the characteristics eq$^n$ of matrix A. and verify that it satifies by A & find $A^{-1}$.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\boxed{|A - \lambda I| = 0}$$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$C_2 \to C_2 + C_3 = C_{23}$$

$$C_{23}(1) \begin{vmatrix} 2-\lambda & 0 & 1 \\ -1 & 1-\lambda & -1 \\ 1 & 1-\lambda & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left[(1-\lambda)(2-\lambda) - (-1)(1-\lambda)\right] + 1\left[(-1(1-\lambda) - 1(1-\lambda)\right]$$

$$(2-\lambda)\left[2 - \lambda - 2\lambda + \lambda^2 + (1-\lambda)\right] + -1 + \lambda - (1-\lambda)]$$

$$\cancel{(2-\lambda)\left[2 - 3\lambda + \lambda^2 + 1 - \lambda + -1 + \lambda - 1 + \lambda\right.}$$

$$(2-\lambda)\left[\lambda^2 - 2\lambda + 1\right)$$

$$\cancel{2\lambda^2 - 4\lambda + 2 - \lambda^3 + 2\lambda^2 - \lambda = 0}$$

$$-\lambda^3 + 4\lambda^2 - 5\lambda$$

$$(2-\lambda)\left[\lambda^2 - 4\lambda + 2\right] + \left[2\lambda - 2\right] = 0$$

P. No.:

The charac. eq.$^n$ $\boxed{\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0}$

Put $\boxed{\lambda = A}$

$$A^3 - 6A^2 + 9A - 4 = 0$$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -6 & 6 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Put the values

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence Proved..

$$\left[A\cdot A^{-1} = I, \quad A\cdot I = A\right]$$

Date:
P.No.:

Now Find $A^{-1}$

Multiply eq. ① by $A^{-1}$, on both side

$$A^2-6A+9I-4A^{-1}=0$$

$$4A^{-1} = [A^2-6A+9I]$$

$$4A^{-1} = \begin{bmatrix}6&-5&5\\-5&6&-5\\5&-6&6\end{bmatrix} - 6\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix} + 9\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$4A^{-1} = \begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix}3/4&1/4&-1/4\\1/4&3/4&1/4\\-1/4&1/4&3/4\end{bmatrix}}$$

$$A^3-6A^2+9A-4=0$$

$$A^2(A\cdot A^{-1}) - 6A(A\cdot A^{-1}) - 9A\cdot A^{-1} - 4A^{-1} = 0$$

$$A(A\cdot I)$$

$$A^2 - 6A - 9A - 4A^{-1} = 0$$

$4A^{-1}$

**Que.** Find the characteristics of matrix A and verify that it satisfies by A and find $A^{-1}$.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0$$

$1-\lambda((2-\lambda)(3-\lambda) - 1\times 0) - 0(\quad) + 2(0-2(2-\lambda$

$1-\lambda(6-2\lambda-3\lambda+\lambda^2) + 2(-4+2\lambda) = 0$

$(1-\lambda)(\lambda^2-5\lambda+6) - 8 + 4\lambda = 0$

$\lambda^2-5\lambda+6-\lambda^3+5\lambda^2-6\lambda - 8+4\lambda = 0$

$-\lambda^3+6\lambda^2-11\lambda+4\lambda-2 = 0$

$\lambda^3-6\lambda^2+7\lambda+2 = 0$

$$\boxed{A^3-6A^2+7A+2I = 0}$$

Multiply both side by $A^{-1}$

$A^2 A\cdot A^{-1} - 6A\cdot A\cdot A^{-1} + 7A\cdot A^{-1} + 2A^{-1} = 0$

$A^2 I - 6AI + 7I + 2A^{-1} = 0$

$A^2 - 6A + 7I + 2A^{-1} = 0$

~~$A^2 - 6A + 7$~~

~~$A^2 - 7A + A + 7$~~

P. No. :

$$2A^{-1} = -A^2 + 6A - 7I$$

$$A^2 = A \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$2A^{-1} = \begin{bmatrix} -5 & 0 & -8 \\ -2 & -4 & -5 \\ -8 & -0 & -13 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2A^{-1} = \begin{bmatrix} -5 & 0 & -8 \\ -2 & -4 & -5 \\ -8 & -0 & -13 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 12 \\ 0 & 12 & 6 \\ 12 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$$

$$2A^{-1} = \begin{bmatrix} -6 & 0 & 4 \\ -2 & 1 & 1 \\ 4 & 0 & -2 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -1 & 1/2 & 1/2 \\ 2 & 0 & -1 \end{bmatrix}}$$ Ans.

$$A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$